

# Hunting for the $X_b$ via Radiative Decays

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In this paper, we study radiative decays of  $X_b$ , the counterpart of the famous  $X(3872)$  in the bottomonium-sector as a candidate for meson-meson molecule, into the  $\gamma\Upsilon(nS)$  ( $n = 1, 2, 3$ ). Since it is likely that the  $X_b$  is below the  $B\bar{B}^*$  threshold and the mass difference between the neutral and charged bottom meson is small compared to the binding energy of the  $X_b$ , the isospin violating decay mode  $X_b \rightarrow \Upsilon(nS)\pi^+\pi^-$  would be greatly suppressed. This will promote the importance of the radiative decays. We use the effective Lagrangian based on the heavy quark symmetry to explore the rescattering mechanism and calculate the partial widths. Our results show that the partial widths into  $\gamma\Upsilon(nS)$  are about 1 keV, and thus the branching fractions may be sizeable, considering the fact the total width may also be smaller than a few MeV like the  $X(3872)$ . These radiative decay modes are of great importance in the experimental search for the  $X_b$  particularly at hadron collider. An observation of the  $X_b$  will provide a deeper insight into the exotic hadron spectroscopy and is helpful to unravel the nature of the states connected by the heavy quark symmetry.

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## I. INTRODUCTION

In the past decades, there has been great progress in hadron spectroscopy thanks to the unprecedented data sample accumulated by the B factories and hadron-hadron colliders. A number of charmonium-like and bottomonium-like states have been discovered on these experimental facilities so far but not all of them can be placed in the ordinary  $\bar{q}q$  (for reviews, see Refs. [1–4]).

The  $X(3872)$  is the first and perhaps the most renowned exotic candidate. It was first discovered in 2003 by Belle in the  $B^+ \rightarrow K^+ + J/\psi\pi^+\pi^-$  final state [5] and subsequently confirmed by the BaBar Collaboration [6]. Complementary observation is also found in proton-proton/antiproton collisions at the Tevatron [7, 8] and LHC [9, 10]. Though the existence is well established, the nature of the  $X(3872)$  is still ambiguous due to a few peculiar properties. First, compared to typical hadronic widths the total width is tiny. Only an upper bound has been measured experimentally:  $\Gamma < 1.2$  MeV [11]. The mass lies closely to the  $D^0\bar{D}^{*0}$  threshold,  $M_{X(3872)} - M_{D^0} - M_{D^{*0}} = (-0.12 \pm 0.24)$  MeV [12], which leads to speculations that the  $X(3872)$  is presumably a meson-

meson molecular state [13, 14].

These peculiar features have stimulated considerable research interest in investigating the production and decays of the  $X(3872)$  towards understanding its nature. A very important aspect involves the discrimination of a compact multiquark configuration and a loosely bound hadronic molecule configuration. In this viewpoint, it would be also valuable to look for the analogue in the bottom sector, referred to as  $X_b$  following the notation suggested in Ref. [15], as states related by heavy quark symmetry may have universal behaviours. Since the  $X_b$  is expected to be very heavy and its  $J^{PC}$  of is  $1^{++}$ , it is less likely for a direct discovery at the current electron-positron collision facilities, though the Super KEKB may provide an opportunity in  $\Upsilon(5S, 6S)$  radiative decays [16].

In Ref. [17], the production of the  $X_b$  at the LHC and the Tevatron has been investigated, along the same line with the studies on the search for exotic states at hadron colliders [18–24]. It is shown that the production rates at the LHC and the Tevatron are sizeable [17]. On the other hand, the search for the  $X_b$  also depends on reconstructing the  $X_b$ , which motivates us to study the  $X_b$  decays. Since this meson is expected to be far below threshold, the isospin violating decay mode for instance  $X_b \rightarrow \Upsilon \pi^+ \pi^-$  is highly suppressed, and this may explain the escape of  $X_b$  in the recent CMS search [25]. As a consequence, radiative decays of the  $X_b$  will be of high priority, on which we will focus in this paper. As we will show in the following, these modes have sizeable decay widths.

To calculate the radiative decays, we study the intermediate meson loop contributions, which have been one of the important nonperturbative transition mechanisms in various transitions, and their impact on the heavy quarkonium transitions, also referred to as coupled-channel effects, has been noticed for a long time [26–28]. The intermediate meson loops mechanism has been applied to study the production and decays of ordinary and exotic states [29–46] and B decays [47–54], and a global agreement with experimental data is found. Thus this approach may be an effective approach to handle the  $X_b$  radiative decays.

The paper is organized as follows. In Sec. II, we will introduce the formalism used in this work. Based on this framework, numerical results are presented in Sec. III and the summary will be given in Sec. IV.

## II. RADIATIVE DECAYS

The calculation of contributions from the meson loops requests the leading order effective Lagrangian. Based on the heavy quark symmetry, we employ the relevant effective Lagrangian for

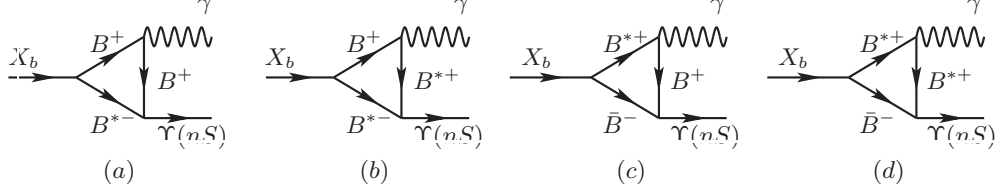


FIG. 1: Feynman diagrams for the radiative decays  $X_b \rightarrow \gamma \Upsilon(nS)$  with the  $B\bar{B}^*$  as the intermediate states.

the  $\Upsilon(nS)$  [54, 55]

$$\begin{aligned} \mathcal{L}_{\Upsilon(nS)B^{(*)}B^{(*)}} = & ig_{\Upsilon BB}\Upsilon_\mu(\partial^\mu B\bar{B} - B\partial^\mu \bar{B}) - g_{\Upsilon B^*B}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\Upsilon_\nu(\partial_\alpha B_\beta^*\bar{B} + B\partial_\alpha \bar{B}_\beta^*) \\ & - ig_{\Upsilon B^*B^*}\{\Upsilon^\mu(\partial_\mu B^{*\nu}\bar{B}_\nu^* - B^{*\nu}\partial_\mu \bar{B}_\nu^*) + (\partial_\mu\Upsilon_\nu B^{*\nu} - \Upsilon_\nu\partial_\mu B^{*\nu})\bar{B}^{*\mu} \\ & + B^{*\mu}(\Upsilon^\nu\partial_\mu \bar{B}_\nu^* - \partial_\mu\Upsilon^\nu \bar{B}_\nu^*)\}, \end{aligned} \quad (1)$$

where  $B^{(*)} = (B^{(*)+}, B^{(*)0})$  and  $\bar{B}^{(*)T} = (B^{(*)-}, \bar{B}^{(*)0})$  correspond to the bottom meson isodoublets.  $\epsilon^{\mu\nu\alpha\beta}$  is the anti-symmetric Levi-Civita tensor and  $\epsilon^{0123} = -1$ . Due to the heavy quark symmetry, the following relationships of the couplings are valid [54, 55]

$$g_{\Upsilon(nS)BB} = 2g_n\sqrt{m_{\Upsilon(nS)}}m_B, \quad g_{\Upsilon(nS)B^*B} = \frac{g_{\Upsilon(nS)BB}}{\sqrt{m_B m_{B^*}}}, \quad g_{\Upsilon(nS)B^*B^*} = g_{\Upsilon(nS)B^*B}\sqrt{\frac{m_{B^*}}{m_B}} \quad (2)$$

where  $g_n = \sqrt{m_{\Upsilon(nS)}}/(2m_B f_{\Upsilon(nS)})$ ;  $m_{\Upsilon(nS)}$  and  $f_{\Upsilon(nS)}$  denote the mass and decay constant of  $\Upsilon(nS)$ , respectively. The decay constant  $f_{\Upsilon(nS)}$  can be extracted from the  $\Upsilon(nS) \rightarrow e^+e^-$ :

$$\Gamma(\Upsilon(nS) \rightarrow e^+e^-) = \frac{4\pi\alpha^2}{27} \frac{f_{\Upsilon(nS)}^2}{m_{\Upsilon(nS)}}, \quad (3)$$

where  $\alpha = 1/137$  is the electromagnetic fine-structure constant. Using the masses and leptonic decay widths of the  $\Upsilon(nS)$  states:  $\Gamma(\Upsilon(1S) \rightarrow e^+e^-) = 1.340 \pm 0.018$  keV,  $\Gamma(\Upsilon(2S) \rightarrow e^+e^-) = 0.612 \pm 0.011$  keV,  $\Gamma(\Upsilon(3S) \rightarrow e^+e^-) = 0.443 \pm 0.008$  keV [11], one can obtain  $f_{\Upsilon(1S)} = 715.2$  MeV,  $f_{\Upsilon(2S)} = 497.5$  MeV, and  $f_{\Upsilon(3S)} = 430.2$  MeV.

We consider the iso-scalar  $X_b$  as a  $S$ -wave molecular state with the positive charge parity given by the superposition of  $B^0\bar{B}^{*0} + c.c$  and  $B^-\bar{B}^{*+} + c.c$  hadronic configurations as

$$|X_b\rangle = \frac{1}{2}[(|B^0\bar{B}^{*0}\rangle - |B^{*0}\bar{B}^0\rangle) + (|B^+B^{*-}\rangle - |B^-B^{*+}\rangle)]. \quad (4)$$

The coupling of  $X_b$  to the bottomed meson is based on the effective Lagrangian

$$\mathcal{L} = \frac{1}{2}X_{b\mu}^\dagger[x_1(B^{*0\mu}\bar{B}^0 - B^0\bar{B}^{*0\mu}) + x_2(B^{*+\mu}B^- - B^+B^{*-\mu})] + h.c., \quad (5)$$

where  $x_i$  denotes the coupling constant.

For a bound state below an  $S$ -wave two-hadron threshold, the effective coupling of this state to the two-body channel is related to the probability of finding the two-hadron component in

the physical wave function of the bound states and the binding energy,  $E_{X_b} = m_B + m_{B^*} - m_{X_b}$  [31, 56, 57]

$$x_i^2 \equiv 16\pi(m_B + m_{B^*})^2 c_i^2 \sqrt{\frac{2E_{X_b}}{\mu}}, \quad (6)$$

where  $c_i = 1/\sqrt{2}$ ,  $\mu = m_B m_{B^*}/(m_B + m_{B^*})$  is the reduced mass.

The magnetic coupling of the photon to heavy bottom meson is described by the Lagrangian [58, 59]

$$\mathcal{L}_\gamma = \frac{e\beta Q_{ab}}{2} F^{\mu\nu} \text{Tr}[H_b^\dagger \sigma_{\mu\nu} H_a] + \frac{eQ'}{2m_Q} F^{\mu\nu} \text{Tr}[H_a^\dagger H_a \sigma_{\mu\nu}], \quad (7)$$

with

$$H = \left( \frac{1 + \not{p}}{2} \right) [\mathcal{B}^{*\mu} \gamma_\mu - \mathcal{B} \gamma_5], \quad (8)$$

where  $Q = \text{diag}\{2/3, -1/3, -1/3\}$  is the light quark charge matrix,  $\beta$  is an unknown parameter and  $Q'$  is the heavy quark electric charge (in units of  $e$ ). In the nonrelativistic constituent quark model  $\beta \simeq 3.0 \text{ GeV}^{-1}$ , which has been adopted in the study of radiative  $D^*$  decays [59]. Note heavy quark symmetry ensures that  $\beta$  is the same in the  $b$  and  $c$  systems, so we take the same value as Ref. [59]. The first term is the magnetic moment coupling of the light quarks, while the second one is the magnetic moment coupling of the heavy quark and hence is suppressed by  $1/m_Q$ .

The decay amplitudes for the transitions in Fig. 1 can be expressed in a generic form in the effective Lagrangian approach as follows,

$$M_{fi} = \int \frac{d^4 q_2}{(2\pi)^4} \sum_{B^* \text{ pol.}} \frac{V_1 V_2 V_3}{a_1 a_2 a_3} \mathcal{F}(m_2, q_2^2) \quad (9)$$

where  $V_i$  and  $a_i = q_i^2 - m_i^2$  ( $i = 1, 2, 3$ ) are the vertex functions and the denominators of the intermediate meson propagators. For example, in Fig. 1 (a),  $V_i$  ( $i = 1, 2, 3$ ) are the vertex functions for the initial  $X_b$ , final bottomonium and photon, respectively.  $a_i$  ( $i = 1, 2, 3$ ) are the denominators for the intermediate  $B^+$ ,  $B^{*-}$  and  $B^+$  propagators, respectively. In addition, we introduce a dipole form factor,

$$\mathcal{F}(m_2, q_2^2) \equiv \left( \frac{\Lambda^2 - m_2^2}{\Lambda^2 - q_2^2} \right)^2, \quad (10)$$

where  $\Lambda \equiv m_2 + \alpha \Lambda_{\text{QCD}}$  and the QCD energy scale  $\Lambda_{\text{QCD}} = 220 \text{ MeV}$ . This form factor is supposed to compensate the off-shell effects arising from the intermediate exchanged particle and the non-local effects of the vertex functions [60–62], and phenomenological studies have suggested  $\alpha \sim 2$ . The explicit expression of the transition amplitudes can be found in Appendix (A.6) in Ref. [63], where radiative decays of charmonium are studied extensively based on the effective Lagrangian approach.

### III. NUMERICAL RESULTS

The existence of the  $X_b$  was predicted in both the tetraquark model [64] and hadronic molecular calculations [65–67]. The mass of the lowest-lying  $1^{++} \bar{b}q\bar{b}q$  tetraquark was predicted to be 10504 MeV in Ref. [64], while the mass of the  $B\bar{B}^*$  molecule based on the mass of the  $X(3872)$  is a few tens of MeV higher [66, 67]. In Ref. [66], the mass was predicted to be  $(10580_{-8}^{+9})$  MeV, corresponding to a binding energy of  $(24_{-9}^{+8})$  MeV. These studies have provided a range for the binding energy, for which in the following we will choose a few illustrative values:  $E_{X_b} = (1, 2, 5, 20)$  MeV.

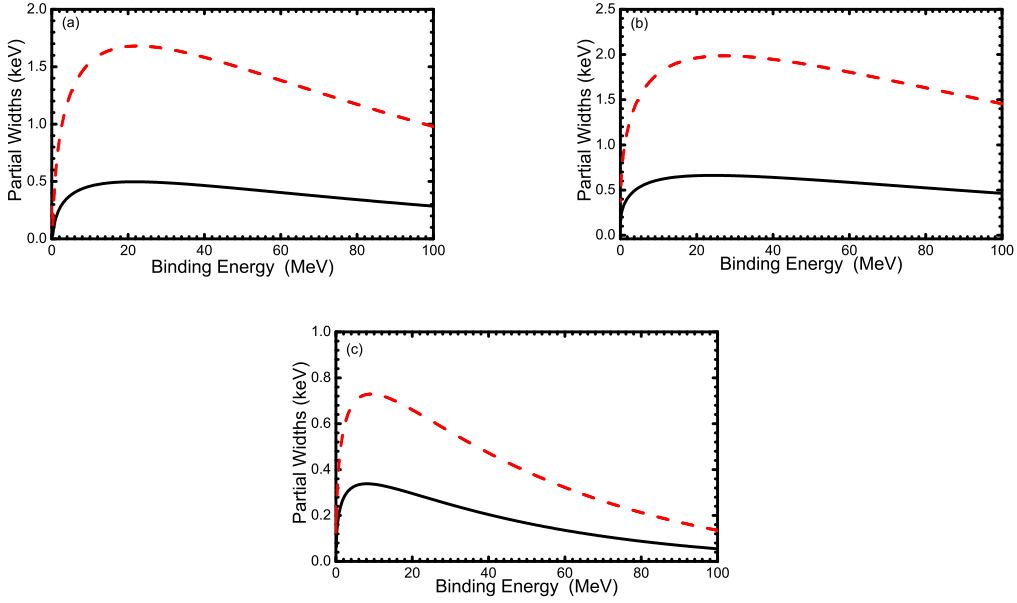


FIG. 2: The dependence of partial widths of  $X_b \rightarrow \gamma\Upsilon(1S)$  on the  $E_{X_b}$  with  $\alpha = 2.0$  (solid lines) and  $\alpha = 3.0$  (dashed lines), respectively. Panels (b) and (c) corresponds to the ones in the  $X_b \rightarrow \gamma\Upsilon(2S)$  and  $3S$ , respectively.

TABLE I: Predicted partial widths (in unit of keV) of the  $X_b$  decays. The parameter in the form factor is chosen as  $\alpha = 2.0$  and  $\alpha = 3.0$ .

	$\alpha = 2.0$			$\alpha = 3.0$		
	$\gamma\Upsilon(1S)$	$\gamma\Upsilon(2S)$	$\gamma\Upsilon(3S)$	$\gamma\Upsilon(1S)$	$\gamma\Upsilon(2S)$	$\gamma\Upsilon(3S)$
$E_{X_b} = 1$ MeV	0.12	0.34	0.22	0.41	0.96	0.46
$E_{X_b} = 2$ MeV	0.19	0.42	0.28	0.62	1.18	0.57
$E_{X_b} = 5$ MeV	0.28	0.53	0.33	0.92	1.53	0.70
$E_{X_b} = 20$ MeV	0.36	0.66	0.30	1.20	1.96	0.66

Choosing two values for the cutoff parameter  $\alpha$ , we have predicted the partial decay widths

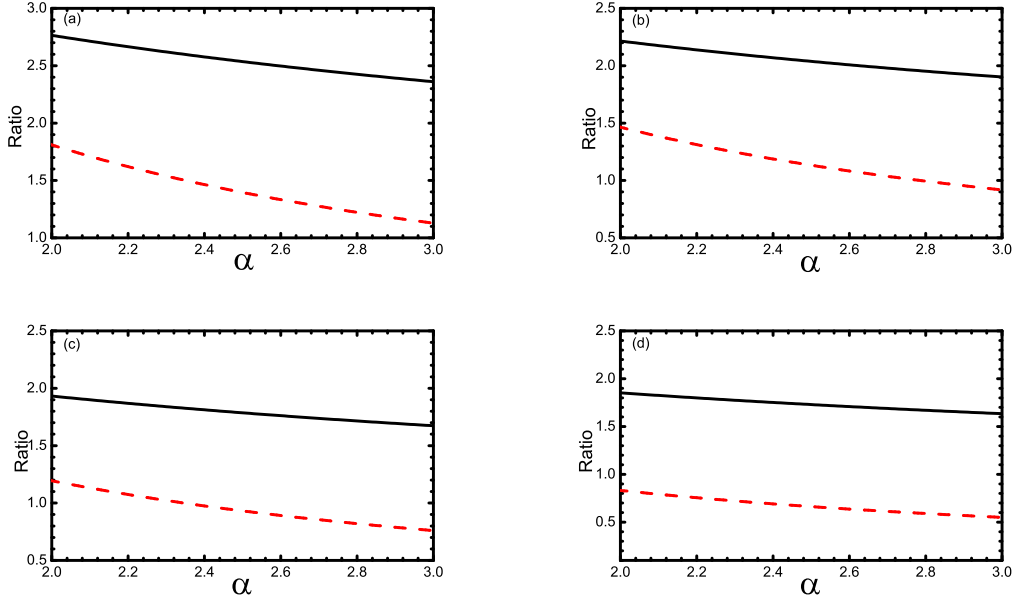


FIG. 3: (a) The  $\alpha$ -dependence of the ratios of  $R_1$  (solid line), and  $R_2$  (dashed line) defined in Eq. (11) with  $E_{X_b} = 1$  MeV. (b), (c), and (d) corresponds to  $E_{X_b} = 2$  MeV, 5 MeV, and 20 MeV, respectively.

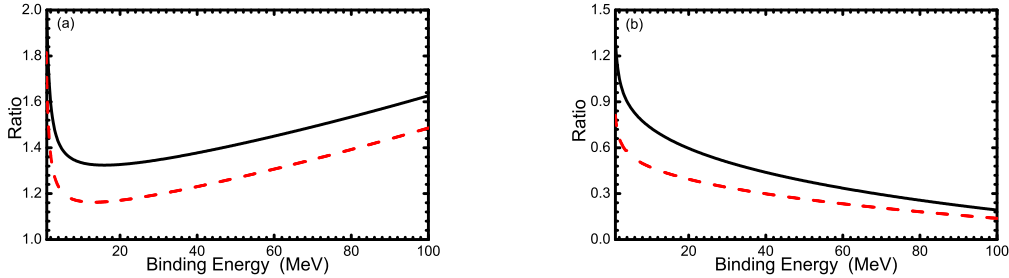


FIG. 4: (a) The ratio  $R_1$  defined in Eq. (11) in terms of the  $E_{X_b}$  with  $\alpha = 2.0$  (solid line) and  $\alpha = 3.0$  (dashed line). (b) The same notation with (a) except for  $R_2$  defined in Eq. (11).

and the numerical results are collected in Table I. From this table, we can see that the widths for the  $X_b$  radiative decays are about 1 keV. It is noteworthy to recall that the upper bound for the  $\Gamma(X(3872))$  is 1.2 MeV [11]. If the  $X_b$  were similarly narrow, our results would indicate a sizeable branching fractions, at least  $10^{-3}$ , for these radiative decay modes.

In Fig. 2, we present the partial widths for the  $X_b \rightarrow \gamma\Upsilon(1S)$  (panel a),  $\gamma\Upsilon(2S)$  (panel b), and  $\gamma\Upsilon(3S)$  (panel c) in terms of the  $E_{X_b}$  with  $\alpha = 2.0$  (solid lines) and 3.0 (dashed lines), respectively. The uncertainties caused by the cutoff parameter indicate our limited knowledge on the applicability of the effective Lagrangian. However fortunately the  $\alpha$  dependence of the partial widths are not drastically sensitive, which indicates a reasonable cutoff of the ultraviolet

contributions by the empirical form factors. In this figure, there exists an evident enhancement structure around  $E_{X_b} = 20$  MeV resulting from the cusp effect. As can be seen from this figure, this enhancement structure is independent of the cutoff parameter  $\alpha$ .

It would be interesting to further clarify the uncertainties arising from the introduction of the form factors by studying the ratios between different partial decay widths. We define the following ratios

$$R_1 = \frac{\Gamma(X_b \rightarrow \gamma\Upsilon(2S))}{\Gamma(X_b \rightarrow \gamma\Upsilon(1S))}, \quad R_2 = \frac{\Gamma(X_b \rightarrow \gamma\Upsilon(3S))}{\Gamma(X_b \rightarrow \gamma\Upsilon(1S))}, \quad (11)$$

which are plotted in Fig. 3 for the dependence on the cutoff parameter and Fig. 4 for the dependence on binding energy. Since the first coupling vertices are the same for those decay channels when taking the ratio, so the ratio only reflects the open threshold effects through the intermediate bottomed meson loops. The ratios are less sensitive to the cutoff parameter, which is a consequence of the fact that the involved loops are the same. As can be seen from this figure, when the cutoff parameter  $\alpha$  increases, the ratios decrease. These predictions can be tested by the experimental measurements in future.

#### IV. SUMMARY

Our understanding of hadron spectroscopy will be greatly improved by studies of exotic states that may defy the conventional models of  $q\bar{q}$  meson spectroscopy, and accordingly great progress has been made in the past decades. One of the most important aspects in the study of exotics is the discrimination of a compact multiquark configuration and a loosely bound hadronic molecule. Such task requests a large amount of efforts on both experimental and theoretical sides in future.

In this work, we have investigated the radiative decays of the  $X_b$ , the counterpart of the famous  $X(3872)$  in the bottomonium-sector as a candidate for meson-meson molecule, into the  $\gamma\Upsilon(nS)$ . Since this state may be far below the  $B\bar{B}^*$  threshold, the isospin violating decay mode  $X_b \rightarrow \Upsilon\pi^+\pi^-$  would be highly suppressed, and stimulate the importance of the radiative decays. We have made use of the effective Lagrangian based on the heavy quark symmetry, and explore the rescattering mechanism. Our results have shown that the partial widths for the  $X_b \rightarrow \gamma\Upsilon(nS)$  are about 1 keV, and thus the branching fractions may be sizeable, taking into account the fact the total width may also be smaller than a few MeV like  $X(3872)$ . This study of radiative decays and the previous work on production rates in hadron-hadron collisions have indicated a promising prospect to find the  $X_b$  at hadron collider in particular the LHC, and we suggest our experimental colleagues to perform an analysis. Such attempt will likely lead to the discovery of the  $X_b$  and thus enrich the exotics garden in the heavy quarkonium sector.

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